Evacuation plan evaluation: Assessment of vehicular evacuation schemes by means of an analytical dynamic traffic model

By
Adam J. Pel*, Michiel C.J. Bliemer

* Delft University of Technology, The Netherlands

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ABSTRACT: An evacuation model is posed, combining a dynamic travel demand component, an adaptive travel choice behaviour component, and a dynamic network loading component. The travel demand component considers the primary choice facing the endangered residents, whether to participate in the evacuation, and if so, when to depart. The travel choice behaviour component considers the secondary choice facing the evacuees, where to seek refuge and by which route to travel towards this safe destination. The network loading component considers both the dynamic traffic and hazard conditions, and propagates the evacuees through the infrastructure network. The proposed model can act on a broad spectrum of hazards, as it uses general features to compute the effects of the hazard on the evacuation. Furthermore, the model structure enables the assessment of various categories of evacuation, ranging from voluntary over recommended to mandatory. And, the behavioural responses of the evacuees towards evacuation instructions are modelled, such that instructions can be followed fully, followed in part, or rejected completely. An illustrative example of a hypothetical evacuation shows the principles and possibilities of the posed evacuation model.

KEY WORDS: Evacuation planning, evacuation behaviour, dynamic traffic modelling

AUTHORS: Adam J. Pel & Michiel C.J. Bliemer

CONTACT: Institute of Transport and Logistics Studies (C37)
The Australian Key Centre in Transport Management
The University of Sydney  NSW  2006  Australia

Telephone: +61 9351 0071
Facsimile: +61 9351 0088
E-mail: itlsinfo@itls.usyd.edu.au
Internet: http://www.itls.usyd.edu.au

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1. Introduction

During the past decades, large storms, mudflows, hurricanes, bush fires, floods, and many other hazards have caused massive economic and social damage as well as loss of life. Since both the severance with which these natural disasters collide with a populated area, and the frequency with which this occurs, are increasing (see [1], [2]), an appropriate response becomes all the more central. The one-sided approach of protecting people by constructing physical barriers at any cost is not unquestionably the most efficient way of dealing with the threat of a hazard. Instead, in some cases, the predictive nature of the hazard is utilized for setting up a warning system to timely relocate the endangered population. In [3] explicit cost-benefit analyses show that by accepting evacuation as an optional response to such hazards, people can be protected at much lower costs.

To make adequate decisions, relating to counteracting or anticipating on hazards, knowledge on the process of evacuation is needed. The costs and benefits of evacuation ought to be understood, to aptly protect property and human life from the imminent conditions. To date, the process of evacuation is still engulfed in uncertainty and unawareness. As discussed in [4], this may result in officials being reluctant to order an evacuation, due to the uncertain and unfamiliar conditions and the financial liability involved. This ambiguity impedes the appropriate authority to make grounded decisions when the moment is there. Better decisions can be made, and society could benefit, if the process of evacuation is better understood and the costs of evacuation reduced. Since a large part of the evacuation process is made up of the physical transportation of people, and in part due to the recent enormous growth in population, the contribution of transport planning to evacuation planning has increased over the years, as shown by [5]. The evacuation transport studies can provide insight into the least amount of time needed for the complete evacuation and develop ways to further minimize this period of time. In other words, knowing until what moment in time, the decision to evacuate can be postponed. The value of such knowledge can be learnt from the failure to timely evacuate New Orleans, as a response to the hurricane Katrina in 2005. The uncertainty, prospect of large costs and chaos, and the risk to human life as a result of the evacuation itself, delayed the order to evacuate. The mandatory evacuation was not directed until 20 hours prior to the impact, which proved to be too late to be effective.

A nice overview of past evacuation models is given in [6], which concludes that the majority of the more recent developed evacuation models attempt to combine the traffic simulation model with geographic information systems, so as to predict the spatial implications of an evacuation. Evacuation schemes can then be assessed and heuristically optimized based on a set of pre-defined criteria. These, mostly macroscopic, models concentrate on the traffic characteristics throughout the evacuation, such as speeds and traffic volumes, and can identify where bottlenecks are likely to occur and calculate the evacuation time. Several of these models use dynamic traffic assignment and simulate the changing network conditions. Overall, the weaknesses of these models are that the travel demand pattern is assumed to be given, the travel choices are based on the traffic conditions, thus giving no credit to the objectives that might arise when faced with the hazard, and the focus lies on evaluating mandatory evacuations, thus not adequately representing the preferences and choice behaviour of the evacuee.
Undoubtedly, evacuation modelling builds on both the behavioural analysis of the evacuees and the simulation of the traffic propagation over time. The evacuation model proposed in this paper recognizes this, and lets the dynamic traffic flows be shaped by the hazard, the authority supervising the evacuation, the evacuees and the traffic propagation conditions. The main advantages of the proposed model, as compared to former approaches of emergency-evacuation models, are that

- the behavioural aspects of the evacuees are included by accounting for the beliefs, desires and intentions of the evacuees, and allowing the disseminated instructions to be followed fully, followed in part, or rejected completely,
- the travel behaviour of the evacuees is represented realistically, by allowing for adaptive en-route destination and route choice,
- various categories of evacuations can be modelled, ranging from voluntary, to recommended, to mandatory evacuation,
- the hazard may strike while the evacuation is on its way and thus hinder and jeopardize the evacuees.

The outline of this paper is as follows. First, in Section 2 the theory on evacuation discusses those facets of the evacuation which determine the evacuating traffic flows. In Section 3 the proposed evacuation model framework is described. Next, in Section 4 the principles of the model are demonstrated in the hypothetical bush fire evacuation on the example network. To end with, conclusions are drawn in Section 5.

2. Theory on evacuation

If we limit the course of the evacuation to the pattern of evacuating traffic flows (thereby excluding factors such as expenses, property destruction and emergency services), then we can say that the course of the evacuation is determined basically by the choice behaviour of the evacuees. That is, the traffic flows are the result of how the endangered residents decide on whether to evacuate, and if so, when to depart, where to evacuate to and which evacuation route to take. How the resident decides to behave, depends primarily on its objectives. Yet, we can see that, for instance, the resident’s options are open to the hazardous conditions, its knowledge is based partially on the information given by third parties or the authority, and it may volunteer or be forced to comply with the evacuation instructions given by the authority. Thus, these elements shape the behaviour of the resident as well. In this section, we reason that the evacuation is determined by the interplay of three distinct facets, namely the hazard, the authority and third parties, and the residents (also referred to as evacuees).

2.1 The hazard

Concerning the hazard, the focus point in evacuation modelling is the similar effects that hazards have on the evacuation. The hazard affects the evacuation process through the spatiotemporal pattern of (life)threatening conditions due to the hazard. Looking at this spatiotemporal pattern, hazards clearly differ as to how they may evolve and affect the surrounding area. Although the differences are apparent, the similarity between the various patterns is the presence of the hazard front, which gradually expands throughout the region. The conditions behind the front then impede the accessibility of parts of the network, and thereby hinder the process of evacuation. The effect is similar, whether
this may be due to the limited visibility (e.g. smoke, heavy rainfall), or physical barrier (e.g. inundation, debris), or risk barrier (e.g. fire, wind blows). Namely, the traffic on the impeded road section will have to reduce their speed, in order to adjust to the new situation. Each further worsening of the situation will bring along a further reduction in the traffic speed. Until eventually, the evacuees can no longer travel along that road section. This way, as the hazard continues, network blockages arise and spread. We model the spatiotemporal pattern of the hazard by the *time-dependent dispersion* and the *destruction force* of the hazard.

### 2.2 The authority and third parties

The costs of evacuating unnecessarily must be weighed against the penalty of not evacuating when justified. In that, the supervising authority decides on the best overall response to the envisaged conditions. When evacuation is warranted, the authority has to select the appropriate evacuation strategy. The whole evacuation process includes aspects of informing the inhabitants and giving warning, organizing transportation for the hospitalized, elderly and otherwise disabled people, providing shelter to the evacuees, safeguarding the deserted area from vandalism, burglary and such, coordinating the emergency services, and so on. In deciding on the make-up of the evacuation scheme, the authority can have various objectives. Reasons for controlling the evacuation process can relate to, for instance

- to optimize the development of the evacuation over time, i.e. maximize the number of saved evacuees and minimize the time necessary to evacuate the region,
- to avoid panic, stress, exposure to risk, evacuation towards endangered areas, excessive (unnecessary) evacuation, and such,
- to minimize the social and economical costs to society – including the costs of injury and death,
- due to the legal responsibility towards the safety of the residents and the safeguarding of private and public property, in case of large-scale disasters.

This list of objectives is not exhaustive. Nor are the listed objectives mutually exclusive. Even though the actions of the authority are society-orientated, the evacuation instructions may benefit residents individually, as well. Namely, the instructions have the potential to reduce the negative overall effects of the unilateral egocentric behaviour of the public, and can lead to higher utilities for a share of the population. The evacuation process can be steered, so as to reach the objectives, in two ways. The authority can provide descriptive information on the conditions, or prescribe a certain procedure through activating the evacuation scheme.

When evacuation is issued, the authority can choose to distribute descriptive information. As well, evacuees can obtain information from other media, not controlled by the authority (e.g. radio, internet, neighbours, etc.). Assuming that all information is true, the influence of the authority and the third parties in distributing descriptive information can be modelled by looking at the *level of information*, as in the amount of information made available to the public.
The evacuation authority may also decide to instruct the population on when the residents are supposed to evacuate, where the evacuees ought to take shelter, and along what route the evacuees are supposed to reach this safe destination. Apart from the content of the instructions, the authority has to decide on the level of enforcement, relating to the effort taken to make sure that the instructions are carried out correctly. So, the authority can give recommendations or binding instructions, where [7] applies this distinction between the different types of instructions to categorize evacuations as voluntary, recommended or mandatory. Often, the course of the evacuation will show a combination of these three categories, as warnings, recommendations, and binding instructions are distributed alongside each other. We will model the evacuation scheme by the sets of evacuation instructions relating to the initiated evacuation time, evacuation destination and evacuation route, and the type of instructions. The maximum number of instruction sets then equals the number of residents. Though, for the sake of feasibility and practicality, a more aggregate regulation level can be applied, where a number of individuals are given the same instruction set.

2.3 The resident

In the field of human psychology, the individual’s behaviour is believed to be determined by the cognitive process of decision-making. A multitude of real life examples in [8] demonstrates that, independent of the type of hazard at hand, a pattern can be seen in the individual’s psychological response to the hazard conditions and instructions. Several attempts have been made to construct a framework describing this psychological decision-making process. While literature shows that various approaches have been undertaken, what most studies agree on is that the individual moves through several psychological phases. The cognitive task starts with (i) information acquisition, followed by (ii) situation assessment, and finally (iii) action execution (first posed by [9]). The amount of information that can be obtained from the environment is too large to be fully assessed, especially considering the limited amount of time available. According to [10] and [11], to avoid this obstacle, the first two phases are undertaken simultaneously. The amount of information is reduced, based on relevance and trustworthiness. This process of filtering the observations makes the task easier, but causes a delay in the appropriate reaction to the nature of the situation. This phenomenon of delayed reaction can be subscribed to the condition termed as perceptual narrowing, observed when an individual is exposed to high levels of stress or information. The dangerous situation spurs the individual to narrow down the breadth of attention, and in turn intensify the efforts made on the single task marked as important. One of the models taking this restricted attention into account is the Belief-Desire-Intention (BDI) behavioural model (discussed in [12] and [13]), which states that the individual’s behaviour depends on a set of beliefs, desires and intentions, and is bounded by the individual’s capabilities. The beliefs relate to the personally formed image on the envisaged state of the conditions. The more familiar the individual is with the situation, or the more information the individual gets, the more correct its beliefs will likely be. The desires state what the individual wishes to achieve or gain, and are represented by the utility associated with the course of action, and follow from the resident’s objectives. The intentions relate to the commitment to carry out a certain course of action. Finally, the capabilities are all the feasible ways in which the individual can respond to the situation. In short, the displayed behaviour aims to satisfy certain needs (desires), though the effectiveness depends heavily on the validity of the
evacuee’s beliefs. If these are in concord with the actual situation, its behaviour maximizes its subjectively weighted utility. If the knowledge of the evacuee is more limited, its behaviour will appear as irrational and random.

2.4 Bi-level authority-resident problem

There is a difference between the authority’s motivation to order and control the evacuation, and the resident’s motivation to evacuate. The two actors assess the situation on different criteria and from a different perspective. The authority attempts to achieve an overall minimum generalized cost to society, whereas the resident attempts to minimize the personal generalized costs. This discrepancy in objectives between the public and the authority may often lead to resistance on the side of the residents (see e.g. [14], [15] and [16]). Since the residents’ behaviour most likely deviates from the proposed or instructed course of behaviour, [17] reasons that the manner in which the public responds to the evacuation instructions becomes an essential consideration while deciding on the appropriate evacuation measures to be taken. In other words, evacuation planning holds that the authority anticipates the expected responsive behaviour of the residents, and adjusts the evacuation instructions accordingly. How likely it is for the evacuation instructions to be accepted and followed depends on the level of overlap between the instructions and the resident’s preferred actions, and the resident’s willingness to comply when the instructions conflict with its personal preferences. This way we will model varying levels of compliance, where the instructions can be accepted fully, accepted in part, or rejected completely.

3. Model framework

3.1 Description of the network

Consider a given network $G$, consisting of nodes $N$ and directed links $A$, where all nodes $n \in N$ are connected and each link $a \in A$ has characteristics, such as the tail-node and head-node, the link length $l_{a}$, the number of lanes $n_{a}$, the free flow speed $f_{a}$, and the link capacity, $c_{a}$. Let the population be assigned to the nodes, such that the number of residents in node $n$ is denoted by $B_{n}$, where $B = [B_{n}]$ is the vector of all residents.

3.2 Description of the hazard

The authority plans the evacuation to the expected probable hazard scenario $H$, defined by the time-dependent dispersion and the destruction force, given by respectively the endangered nodes $R$ and strike times $I$, and the force at the nodes $\Upsilon$ and impediments on the links $\Gamma$. The hazard will spread through part of the given region, where $R$ denotes the set of nodes within that affected part. Let $S$ denote the set of safe destinations, $S \subseteq N \setminus R$. Since only the expected endangered residents are prone to evacuate, in the evacuation model we need only to consider the residents in $[B^{r}]$, $r \in R$. Let $T$ denote the total set of time periods considered in the evacuation model. The time period that the hazard strikes node $r \in R$ is denoted by $t' \in T$, while the vector of all strike times is denoted by $I = [t']$. The hazard force at each node $r \in R$ at time period $t \in T$ will be denoted as $\Upsilon'(t)$, such that $\Upsilon = [\Upsilon'(t)]$. The link impediments, denoted by $\Gamma = [\gamma_{a}(t)]$, describe the impediment on each link $a \in A$ at time period $t \in T$. Typically,
\( \gamma_a(t) \in [0,1] \), where 1 indicates that link \( a \) is fully accessible at time period \( t \), while 0 means it is no longer accessible due to the hazard. Any value between 0 and 1 means that the link has limited accessibility.

Note that we are considering time intervals instead of time instants, implicitly assuming discrete time intervals. Each time interval is assumed to be of size \( \delta \), such that time period \( t \in T \) indicates the time interval \([t-1)\delta, t\delta)\). The smaller this time step \( \delta \), the closer the model approximates the continuous time formulation.

### 3.3 Description of the authority and third parties

The level of information made available at evacuation time period \( k \) relevant to the evacuation participation choice and evacuation time choice, will be denoted as \( \lambda_{\text{part}}(k) \), while \( \lambda_{\text{route}}(t) \) denotes the information at time period \( t \) relevant to the evacuation destination and route choice. This includes the information given by the authority and by the third parties. Typically, \( \lambda_{\text{part}}(k) \in [0,1] \) and \( \lambda_{\text{route}}(t) \in [0,1] \), where 1 indicates that the evacuees’ beliefs are valid, while 0 means that the evacuees are unaware of the current conditions, and their decisions can be seen as random. Any value between 0 and 1 means that the evacuees’ beliefs are partially in concord with the actual conditions.

The authority can give different evacuation instructions to different groups of residents, with instructions on the evacuation participation, evacuation time, evacuation destination, and evacuation route. Let \( M \) denote the number of different evacuation schemes and let \( e_m \in E \) be the evacuation instructions for the group \( m \) residents. Each evacuation scheme \( e_m, m = 1, \ldots, M \), is then described by \( e_m = \{K_m, S_m, P_m, \omega_m\} \), where \( K_m \) is the set of instructed departure time periods, \( S_m \subseteq S \) is the set of instructed safe destinations, and \( P_m \) is the set of instructed routes. Suppose that evacuation instructions \( e_m \) are given to a certain number of residents in node \( r \in R \), denoted by \( B'_m \). Since residents in a certain node \( r \) can never receive exactly the same route to a destination as residents in another node (since the first link will be different and therefore set \( P_m \) will always be different), residents in different nodes cannot receive the exact same instruction set. The sets of residents given different evacuation instructions will be called evacuation classes, which will be used later on in our proposed multiclass dynamic network assignment model. Let \( \omega_m \) indicate the type of instructions for class \( m \). Typically, \( \omega_m \in [0,1] \), where 1 indicates that the instructions are binding, while any value between 0 and 1 means that the instructions can be seen as recommendations, and a higher value indicates a stronger recommendation.

### 3.4 Dynamic travel demand

We assume that the public is unfamiliar with the hazardous conditions, so instead of relying on habit, the individual will show adaptive choice behaviour according to the BDI model discussed in Section 2.3. This implies that the estimated utilities (desires) are continuously updated towards the current beliefs and that the individual will continuously behave according to its currently estimated utilities.

At each time period, the resident has the opportunity to either select the current time period to evacuate in or decide to evacuate at a future time period. This binary
evacuation decision is repeated for each time period until the resident decides to evacuate and thereby simultaneously selects the current evacuation time period. Hence, the evacuation participation choice and the evacuation time choice can be modelled as a joint choice process. Furthermore, due to adaptive choice behaviour this joint choice process can be modelled sequential to, and independent of, the travel choice behaviour, as argued in [18].

Let the observed utility of evacuating at time period $k$ for class $m$ residents at node $r$ be denoted by $W_{m,r}^{\text{evac}}(k)$, and let $W_{m,r}^{\text{stay}}(k)$ be the corresponding observed utility to stay. Assuming that the unobserved components of their utilities are independently and identically extreme value type I distributed, the proportion of the initial population that prefers to evacuate, $Q_{m}^{\text{part}}(k)$, can be computed using the following binary logit model:

\[
Q_{m}^{\text{part}}(k) = \frac{1}{1 + \exp\left(\mu_{\text{part}}^{\text{part}}(k)\left(W_{m,r}^{\text{stay}}(k) - W_{m,r}^{\text{evac}}(k)\right)\right)}, \tag{1}
\]

where \( \mu_{\text{part}}^{\text{part}}(k) = \frac{\lambda_{\text{part}}(k)}{1 - \lambda_{\text{part}}(k)} \).

If the evacuees’ beliefs are valid, when $\lambda_{\text{part}}(k) = 1$, then $\mu_{\text{part}}^{\text{part}}(k) = \infty$, and the proportion that prefers to evacuate depends on the objective utilities to stay and to evacuate. When the evacuees’ beliefs differ from the actual conditions, when $0 \leq \lambda_{\text{part}}(t) < 1$, then $0 < \mu_{\text{part}}^{\text{part}}(t) < \infty$, and thus the randomness in the preferred evacuation proportion varies with the validity of the evacuees’ beliefs.

Let $F_{m}^{r}(k)$ be the node and class specific cumulative travel demand up till evacuation time $k$. Then the travel demand at the first evacuation time $k = 1$ is given by $Q_{m}^{r}(1)B^r_m$. Since the cumulative travel demand is by definition monotonically increasing, for any subsequent evacuation time period $k > 1$, the cumulative travel demand is bounded from below by the preceding time periods. Hence,

\[
F_{m}^{r}(k) = \begin{cases} Q_{m}^{r}(k)B^r_m, & \text{if } Q_{m}^{r}(k) > Q_{m}^{r}(k'), \forall k' \in T | k' < k; \\ F_{m}^{r}(k - 1), & \text{otherwise.} \end{cases} \tag{2}
\]

To stay at home takes in the opportunity to undertake property protection [19]. The objective observable utility to stay $W_{m,r}^{\text{stay}}(k)$, is set equal to $\alpha_0(> 0)$, where $\alpha_0$ is a constant for the opportunity to protect one’s property:

\[
W_{m,r}^{\text{stay}}(k) = \alpha_0. \tag{3}
\]

We assume that the observed utility to evacuate can be modelled linearly as:

\[
W_{m,r}^{\text{evac}}(k) = \alpha_{1}u^{r}(k) + \alpha_{2}\delta\left(t^{r} - k\right)^{t} + \alpha_{3}\rho_{m}^{r}\text{time}(k), \tag{4}
\]

\[\text{Arguably, other attributes could be added to the evacuation utility function. For instance, for the case of bush fires, [20] identifies multiple, other attributes which prove to be statistically significant in the decision to evacuate.}\]
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where \( (x)^+ = \max\{0, x\} \), and \( \rho_m = \frac{\omega_m}{1 - \omega_m} \).

The weighting parameters \( \alpha_1(>0) \) and \( \alpha_2(<0) \) show the utility associated with a change in the hazard force \( \nu'(k) \), respectively the proximity of the hazard front, where the proximity is measured in terms of time until the hazard strikes origin \( r \), given by the strike time period \( \iota \in T \). The compliance with the instructed evacuation time is measured by the time overlap factor, \( \xi^\text{time}_m(k) \). Assuming that the set of instructed departure time periods gives the continuous time interval \( K_m = \left( k_1, k^* \right) \), where \( k'_m = \min\{K_m\} \) and \( k^* = \max\{K_m\} \). Then for \( k < k' \), to comply means to stay, thus the time overlap factor can be set by the time-difference \( \xi^\text{time}_m(k) = \delta(k - k'_m) \), where \( k'_m = k' \). Since the authority is indifferent to the evacuation time \( k \) when \( k' < k < k^* \), the resident can evacuate at \( k \) or postpone the evacuation, therefore \( \xi^\text{time}_m(k) = 0 \). After \( (k^* - 1) \), the evacuation can no longer be postponed and to comply means to evacuate – alternatively, to evacuate belatedly since the instruction is to participate in the evacuation. Thus, in case of \( k > (k^* - 1) \), the time overlap factor can be set by the time-difference \( \delta(k - k^\text{upper}_m) \), where \( k^\text{upper}_m = k^* - 1 \). Hence,

\[
\xi^\text{time}_m(k) = \begin{cases} 
0, & \text{if } k'_m \leq k \leq k^\text{upper}_m, \\
\delta(k - \kappa_m(k)), & \text{otherwise},
\end{cases}
\]

where \( \kappa_m(k) = \arg\min_{k' \in [k'_m, k^\text{upper}_m]} |k - k'| \).

The non-negative weighting parameter\(^2\) \( \alpha \) represents the willingness to comply with the instructed evacuation time, where a higher value indicates the resident is more willing. The parameter \( \rho \) represents the level of enforcement conveyed by the authority and is defined according to the type of instructions. In case of binding instructions, when \( \omega_m = 1 \), then \( \rho = \infty \), which thus outweighs the contribution of the hazard to the utility function, and forces the resident to evacuate at one of the instructed evacuation time periods, \( K_m \). In case of recommended instructions, when \( 0 < \omega_m < 1 \), then \( 0 < \rho_m < \infty \), and thus the relative contribution of the evacuation time instruction varies according to the strength of the recommendations.

### 3.5 Travel choice behaviour

The evacuees have to decide where to seek refuge and via which route to travel towards this safe destination. The selected destination is inherent to the selected route. Vice versa, the possible routes are bounded by the selected destination, in case of there being multiple safe destinations. So, the evacuation destination choice and the evacuation route choice can be modelled simultaneously as a joint choice process. We would like to point out that in our proposed model, evacuees can follow prescribed evacuation routes to prescribed safe destinations, but may also deviate and make en-route decisions to change their route and destination (possibly due to links that become inaccessible due to the hazard).

\(^2\) Alternatively, two separate weighting parameters can be applied to model the willingness to postpone, respectively advance, the evacuation time.
Let $W_{mp}^{nt}(t)$ denote the observed utility for the class $m$ evacuees to select route $p$, from the current node $n$ to safe destination $s$, based on the conditions at time period $t$. Using a path-size logit formulation proposed in [21] in order to take path overlap into account, the route flow proportions, $\chi_{mp}^{nt}(t)$, can be computed using the following multinomial logit model:

$$\chi_{mp}^{nt}(t) = \frac{\exp(\mu_{\text{route}}(t)(W_{mp}^{nt}(t) + \pi \ln \psi_{p}^{nt}))}{\sum_{s' \in S} \sum_{p' \in P^{nt}} \exp(\mu_{\text{route}}(t)(W_{mp}^{nt}(t) + \pi \ln \psi_{p'}^{nt}))},$$

where $\mu_{\text{route}}(t) = \frac{\lambda_{\text{route}}(t)}{1 - \lambda_{\text{route}}(t)}$,

and $P^{nt}$ is the route choice set from node $n$ to safe destination $s$. If the evacuees’ beliefs are valid, when $\lambda_{\text{route}}(t) = 1$, then $\mu_{\text{route}}(t) = \infty$, and the route flow proportions depend on the relative hierarchy of the objective route utilities. When the evacuees’ beliefs differ from the actual conditions, when $0 \leq \lambda_{\text{route}}(t) < 1$, then $0 \leq \mu_{\text{route}}(t) < \infty$, and the randomness in the route flow proportions varies with the validity of the evacuees’ beliefs. The path-size factor $\psi_{p}^{nt}$, taken from [21], corrects for the assumption of independent choice alternatives, where $\pi$ is the scale parameter for the path-size factor. The path-size factor represents the correction term to the route utility for the partial overlap with other routes in the route choice set, and is set as

$$\psi_{p}^{nt} = \sum_{a \in P} \left( \frac{1}{N_{a}^{nt}} \right),$$

where $N_{a}^{nt}$ is the number of routes from $n$ to $s$ that use link $a$. Hence, a smaller path-size factor represents a less unique route and reduces the likelihood of selecting that individual route alternative.

Let all the safe nodes be equally safe and provide sufficient shelter and support (e.g. medical help, food). Then the hazard has no influence on the destination choice, other then that it may cause some destinations to become infeasible, as all the routes leading to the destination are blocked. We assume that the observed route utility can be modelled linearly as

$$W_{mp}^{nt}(t) = \frac{\tau_{p}^{nt}(t)}{1 - \omega_{m}},$$

and $\tau_{p}^{nt}(t)$ denotes the route travel time. The level of compliance with the evacuation instructions is measured by the rate of overlap between the destination, respectively route, and the set of destinations, respectively routes, initiated by the evacuation scheme. Since the destination is indivisible, the destination overlap factor $z_{m,\text{dest}}^{s}$, is the binary indicator stating whether safe destination $s$ belongs to the set of instructed destinations $S_{m}$.

Arguably, other attributes could be added to the route utility function. For instance the observed risk exposure on the route could be added, by looking at the hazard force and the distance to the hazard, aggregated over the total length of the route.
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\begin{equation}
\xi_{s,\text{dest}}^m = \begin{cases} 
1, & \text{if } s \in S^m, \\
0, & \text{if } s \notin S^m.
\end{cases}
\end{equation}

If the destination overlaps with the set of instructed destinations, then travelling towards the destination results in following the evacuation instructions, and therefore the more preferable this safe destination becomes. The route overlap factor \( \xi_{\text{route}}^m \), is defined as the fraction of the route length that intersects with the set of instructed routes \( P_m \),

\begin{equation}
\xi_{\text{route}}^m = \frac{\sum_{a \in A_{\text{mp}}} \ell_a}{\ell_p},
\end{equation}

where \( \ell_a \) is the length of link \( a \), \( \ell_p \) denotes the length of route \( p \), and \( A_{\text{mp}} \) is the set of links, where each link is both in route \( p \) and one of the instructed evacuation routes \( P_m \). The larger the route overlap factor, the closer the evacuee is to following the instructed route, and therefore the more preferable the route becomes. The non-negative weighting parameters \( \beta_1 \) and \( \beta_2 \) represent the willingness to comply with the instructed evacuation destination, respectively instructed evacuate route, where a higher value indicates the resident is more willing. The parameter \( \rho_m \) represents the level of enforcement and is set similar to the same parameter in Equation (4).

3.6 Dynamic Network Loading

The dynamic network loading consists of two components, namely on node level the traffic flow is distributed over the downstream links according to the adaptive en-route travel choice behaviour of the evacuees, and on link level the traffic flow is propagated through the network according to the dynamic traffic and hazard conditions.

Node model

The traffic flows at the nodes are stated mathematically as

\begin{equation}
f_m^n(t) + \sum_{a \in A_{\text{in}}^n} v_{am}(t) = y_m^n(t) + \sum_{a \in A_{\text{out}}^n} u_{am}(t), \quad \forall m \in M, \forall n \in N, \forall t \in T.
\end{equation}

All the generated traffic is stored on the links. Therefore, at each time period, the travel demand rate \( f_m^n(t) \) plus the summed link outflow rates \( v_{am}(t) \) for all incoming links into node \( n \) (indicated by set \( A_{\text{in}}^n \)), equals the arrival flow rate \( y_m^n(t) \) plus the summed link inflow rates \( u_{am}(t) \) for all outgoing links \( a \in A_{\text{out}}^n \). The class specific travel demand rate (in terms of vehicles, not evacuees) from \( n \) is

\begin{equation}
f_m^n(t) = \begin{cases} 
\frac{F_m^n(t) - F_m^n(t-1)}{\phi_\delta}, & \text{if } n \in R; \\
0, & \text{otherwise},
\end{cases}
\end{equation}
where $\phi$ is the average number of evacuees in a single vehicle. We will define the cumulative class specific arrivals (in evacuees) in a safe destination as

$$Y_m^n(t) = \sum_{i \in S} y_m^n(t')\phi \delta', \quad \forall n \in S.$$  \hspace{1cm} (13)

The link inflow rates $u_{am}(t)$ can be decomposed into flow rates following a specific route $p$ from node $n$ to a specific destination $s$, denoted by $u_{amp}^{as}(t)$. By definition it holds that

$$u_{am}(t) = \sum_{i \in S} \sum_{p \in P} u_{amp}^{as}(t). \hspace{1cm} (14)$$

These route and destination specific link inflow rates $u_{amp}^{as}(t)$ are computed using the route and destination choice proportions from Equation (6) applied to the evacuees currently travelling from node $n$,

$$u_{amp}^{as}(t) = \chi_{mp}(t) \left( f_m^n(t) + \sum_{a \in d'_n(n)} v_m^n(t) \right). \hspace{1cm} (15)$$

The link outflow rate depends on the traffic propagation described by the link model described next.

**Link model**

If the network is empty at the start of the evacuation, then the cumulative link outflow follows the cumulative link inflow, where the time-delay equals the experienced link travel time $\tau_a(t)$, as a function of the link speed, $b_a(t)$. To compute the link speed we apply an adapted Daganzo’s speed-density function (for original formulation, see [22]). The link speed function is plotted in Figure 1, and stated mathematically as

$$b_a(t) = \begin{cases} 
\gamma_a(t)b_a^f, & \text{if } 0 \leq d_a(t) \leq d_a^c; \\
\frac{\gamma_a(t)c_a}{d_a(t)}, & \text{if } d_a^c < d_a(t) \leq \tilde{d}; \\
\gamma_a(t)q_d, & \text{if } d_a(t) > \tilde{d},
\end{cases} \hspace{1cm} (16)$$

where $d_a^c = \frac{c_a}{n_a b_a^f}$, and $\tilde{d} = d^j - \frac{n_a q_d}{c_a} (d^j - d_a^c)$,

where $c_a$, $n_a$, $b_a^f$, $d_a^c$, and $d^j$ are the link capacity, the number of lanes, the free-flow link speed and the critical link density on link $a$, and the jam density independent of the link, respectively. Although the link speed may become zero at certain time instants, clearly the average link speed over an aggregated time period is positive, as the link flow will surely be able to leave the link eventually. Therefore, we assume a minimum link flow rate $q_a$, in case of high link densities $d_a(t) > \tilde{d}$, where $\tilde{d}$ is set in Equation (16) to ensure a continuous link speed function. The adapted link speed function allows for variable link accessibility, $\gamma_a(t)$, and is plotted in Figure 1 for $\gamma = 1$ and $\gamma = 0.5$. 


When the link is inaccessible, the link speed becomes zero and no flow can exit the link anymore. The link travel time for the evacuees entering link $a$ at time period $t$ is

$$\tau_a(t) = \frac{\ell_a}{b_a(t)}. \quad (17)$$

![Adapted Link Speed Function](image)

**Fig 1: Adapted Link Speed Function**

Let $X_a(t)$ denote the link load at the beginning of time period $t$. The link density can then be calculated as

$$d_a(t) = \frac{X_a(t)}{n_a\ell_a}, \quad \text{where} \quad X_a(t) = \sum_{m \in M} \left(U_{am}(t) - V_{am}(t)\right). \quad (18)$$

where $U_{am}(t)$ and $V_{am}(t)$ are class specific cumulative link inflows and outflows, respectively. The cumulative link inflow is by definition

$$U_{am}(t) = \sum_{t' \in \Omega_{st}} u_{am}(t')\delta. \quad (19)$$

In order to compute the cumulative outflow at time $t$, we determine which entering flows have already left the link. If a vehicle enters link $a$ at time period $t'$, which is time instant $t'\delta$, then it will exit the link at time instant $t'\delta + \tau_a(t')$ (but only if the link is still accessible at exit time $t$, i.e. if $\gamma_a(t) > 0$). Therefore, the cumulative outflow is given by

$$V_{am}(t) = \sum_{t' \in \Omega_{at}(t)} u_{am}(t')\delta, \quad \text{with} \quad \Omega_{at}(t) = \left\{t' \in T \mid t'\delta + \tau_a(t') \leq t\delta, \gamma_a(t) > 0\right\}. \quad (20)$$

Finally, the class specific link outflow rates for the node model in Equation (11) are

$$v_{am}(t) = \frac{V_{am}(t) - V_{am}(t-1)}{\delta}. \quad (21)$$
3.7 Evacuation termination

The hazard hinders the evacuation by reducing the link accessibility and may cause network blockages. A route is accessible if the link impediments of all the links in the route are larger than zero. Let $P^n = \{P^n\}$ denote the total set of routes from node $n$ to any safe destination. So, the set of accessible evacuation routes from node $n$ is

\[ \Xi^n(t) = \{ p \in P^n | \gamma_a(t) > 0, \forall a \in p \}. \]  \hfill (22)

The evacuation continues only as long as a number of evacuees can possibly reach a safe destination, i.e. as long as $\Xi^s(t) \neq \emptyset$. Let, at time period $t$, $\eta^{res}(t)$ denote the remaining population at the origins with an accessible evacuation route, and $\eta^{evac}(t)$ denote the number of evacuees on the network with an accessible evacuation route. Then,

\[ \eta^{res}(t) = \sum_{m \in M} \sum_{r \in R(t)} \left(B'_m - F'_m(t)\right), \quad \text{with} \quad \bar{R}(t) = \{ r \in R | \Xi^s(t) \neq \emptyset \}, \]  \hfill (23)

\[ \eta^{evac}(t) = \varphi \sum_{n \in N} \sum_{a \in A^n(n)} X_a(t), \]  \hfill (24)

with $\bar{A}^n(n,t) = \{ a \in A^m(n) | \Xi^s(t) \neq \emptyset, \gamma_a(t) > 0 \}$.

Clearly, the evacuation has ended when both $\eta^{res}(t)$ and $\eta^{evac}(t)$ are equal to zero.

4. Example

4.1 Case description

To illustrate the principles and possibilities of the proposed evacuation model, we apply the evacuation model to the hypothetical scenario of a bush fire approaching a populated area. Figure 2 shows the example network, which consists of 26 directed links and 10 nodes. Each directed link has certain characteristics as described in Section 3.1. The link lengths of the horizontal links, from left to right, are 2.5 km, 4.0 km and 5.0 km, respectively. The vertical links are 3.0 km, and the diagonal links on the right-hand side are 6.0 km. All links have one single lane, free flow speed $b_{f} = 80 \text{ km}\cdot\text{h}^{-1}$, link capacity $c_m = 2000 \text{ veh}\cdot\text{h}^{-1}$, and minimum link flow rate $q_a = 200 \text{ veh}\cdot\text{h}^{-1}$. The jam density is chosen as $d_j = 150 \text{ veh}\cdot\text{km}^{-1}\cdot\text{ln}^{-1}$. The set of nodes contains 2 origins, 2 safe destinations and 6 (endangered) intersections. The initial population at each origin is 2000 and the number of evacuees per vehicle is $\varphi = 2$, such that 1000 vehicles depart from each of the origins.

Let the bush fire start 3.5 km southeast of origin 2, and the dispersion over time be schematized in the figure by the eccentric dashed ellipses with decreasing line thickness, where we let the fire advance with a speed of 2 km\cdot\text{h}^{-1} perpendicular to the fire front. We assume a constant destruction force of 2000 kW\cdot\text{h}^{-1}\cdot\text{m}^{-2}, and let the link be impeded proportionally to the proximity of the bush fire when the fire comes within 300 meters of the link, and blocked when the fire engulfs the link.
Evacuation plan evaluation: Assessment of vehicular evacuation schemes by means of an analytical dynamic traffic model

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Fig 2: Case Scenario

The levels of information for all time periods are \( \lambda^{\text{part}}(k) = 0.5 \), and \( \lambda^{\text{route}}(t) = 0.98 \), with the path-size scale parameter set according to \( \lambda^{\text{route}}(t) \) as \( \pi = 1/\lambda^{\text{route}}(t) \). Note that these values are irrelevant in the mandatory evacuation, as long as they are positive. The proposed evacuation model enables us to simulate both (i) the voluntary evacuation and (ii) the mandatory evacuation with binding instructions. This way, the performance of the mandatory evacuation scheme can be assessed, using the voluntary evacuation as a benchmark. The modelled evacuation scheme instructs the evacuees to travel directly from their origin to the opposite safe destination, where the instructed departure time period is set to maximize the link flow rates. After approximately an hour of evacuation, the instructed evacuation routes are redirected one block upwards. The route flow from origin 2 is redirected to avoid the fire in time, while the origin 1 route flow is redirected to keep the route flows separated, and thus maximize the link flow rates. The instructions are binding, hence for all evacuation classes \( \omega_m = 1 \).

If we assume that for a bush fire with a force below 200 kW·h\(^{-1}\)·m\(^{-2}\) the residents can easily cope with the conditions, and for a bush fire of 4000 kW·h\(^{-1}\)·m\(^{-2}\) the entire population will evacuate at least 2 hours prior to when the fire front will strike. Then, Equations (3) and (4) show that the turning points in the voluntary evacuation, where \( W_m^{evac}(k) = W_m^{stay}(k) \), give

\[
\alpha_0 = \alpha_1 \cdot 200 + \alpha_2 \cdot 0, \quad \text{and} \quad \alpha_0 = \alpha_1 \cdot 4000 + \alpha_2 \cdot 2.
\]

Since we are interested in the relative utilities, we may assume any positive value for \( \alpha_0 \) and set \( \alpha_1 \) and \( \alpha_2 \) accordingly. For the compliance parameters \( \alpha_3 \), \( \beta_1 \) and \( \beta_1 \) in the Equations (4) and (8) any positive value can be taken, since the scale of the values matter only in the recommended evacuation. Finally, we would like to point out that the (weighting) parameter values in this example are chosen for illustrative purposes only.
4.2 Results

The simulation algorithm for the proposed evacuation model is outlined in the appendix, and programmed in Matlab. Applying a time step of \( \delta = 20 \) seconds in the simulation of the described example (leading to approximately 450 time period iterations), the PC running time on a Windows XP computer with Pentium 4 is less than one minute. Clearly any time step can be chosen as long as \( \delta \leq \min_{\{\text{veh,}A\}} \{\tau_o\} \), where a smaller time step, approximates more the continuous time formulation.

In Figure 3, the cumulative travel demand and the cumulative arrivals are graphed for each of the residential origins. At the end of the evacuation, all residents have started evacuation as \( B' = F'(T), \ \forall r \in R \). Yet, a number of evacuees have failed to reach their destination. On the right-hand side of the figure, \( Y^1 \) and \( Y^2 \) denote the destination specific cumulative arrivals at the end of the evacuation. The difference between the total arrivals and the total travel demands is the origin specific numbers of evacuees, which are trapped on the network, where the total number of trapped evacuees is \( \sum_{n \in N} (F'(T) - Y'(T)) \).

By comparing the model results for both the voluntary and mandatory evacuation, the effect of the evacuation scheme is clear. The evacuation time instruction benefits the origin 1 population, while the population in origin 2 benefits mainly from the evacuation destination and route instruction. Recall that the instructed departure time is set to maximize the link flow rates. Hence, the linear cumulative travel demands (for a uniform, maximal travel demand rate). In the mandatory evacuation first-in-first-out holds on origin-destination level, since all evacuees are assigned to a single (instructed) evacuation route. Thus the time length between the cumulative arrivals \( Y(t + \tau_p) \), and cumulative travel demand \( F(t) \), represents the route travel time \( \tau_p \), when \( Y(t + \tau_p) = F(t) \). Recall that the initial instructed route is the shortest route, until the instructed evacuation route shifts upward before the bush fire blocks the lower network links. Hence, the sudden horizontal displacement in the cumulative arrivals curve, indicating the increase in route travel time for the new instructed evacuation route. Note that in the voluntary evacuation the time length between the cumulative arrivals curve and cumulative travel demand curve does not represent the route travel time, since these values are aggregated over multiple used routes. In the mandatory evacuation the instructions maximize the link flow rates, such that the link speed and link density equal respectively the free-flow speed and critical link density on the used links. In the voluntary evacuation, the unilateral egocentric behaviour of the evacuees leads to higher travel demand rates and higher link densities on the shortest route between the origins and destinations. In this case, the link speeds are lower due to the constrained conditions.

In the example case, the evacuation scheme was constructed to solve the negative effects of network blockages. In other more complex scenarios, the simulation of the voluntary evacuation can be used to identify the effect of network blockages, bottlenecks in the network, heedless travel behaviour, and such, which can then be dealt with in the instructed evacuation scheme.
5. Conclusions

When an authority plans an evacuation, it decides on the most appropriate evacuation scheme. Like most evacuation models to date, the proposed model can evaluate different evacuation schemes, to let the authority eventually resolve upon the most appropriate scheme. However, a main advantage of the proposed model is that we can simulate the effect of network blockages due to the hazard. This gives the authority more insight into the probable evacuation process and shows how sensitive the performance of an evacuation scheme is when applied to multiple hazard configurations. A second main advantage is that different types of evacuation instructions can be given. This allows us to simulate the broad range from a voluntary evacuation, to a recommended or mandatory evacuation, with a single model. Hence, a broader range of different evacuation schemes can be evaluated. And last, another important advantage is that we can model different behavioural responses of the public,
such as departure time choice and (en-route or pre-trip) destination and route choice, thus considering the beliefs, desires, intentions and choice alternatives of the evacuees. Finally, we may well conclude that all the mentioned advantages make the simulation of the evacuation more realistic.

References


